Plurals, order, and repetition

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Introduction

Many philosophers and logicians now hold the following view. In a sentence like *The kids met*, the plural subject refers to several kids at once, and the predicate *met* applies to them collectively. Plural reference and collective predication are logically primitive.

Now, certain sentences seem to involve something different:

John, the kids, and Mary got gold, silver, and bronze, respectively.
The last Roland Garros winners are David, Øystein, Øystein, David, and Peter, in that order.
Zero, one, and one are consecutive Fibonacci numbers.

Here, order and possibly repetition matter, contrary to what happens with ordinary plural sentences.

How should we account for such cases? In particular, should we agree with Hewitt (2012) that reference to some things in an order should also be taken to be logically primitive?

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I) Data

a) Examples

Cases where order, and possibly repetition, play a role

With the adverb *respectively:*
Clarence, Florence, and Terrence got the three highest grades, respectively. (Kay)
The three brightest students got the three highest grades, respectively. (Kay)
The four best poems were written, respectively, by John, Bill, John, and Mary.
John, the kids, and Mary got gold, silver, and bronze, respectively.
Sue, Karen, and Bob jog, drive, and jog, respectively. (Gawron & Kehler)
#Sue, Karen, and Bob jog and drive, respectively. (Gawron & Kehler)
Though the Trail Blazers won this series in six games from Phoenix, they were far from dominant. Their margins of victory were 2 points, 1 point, 6 points, and 3 points, respectively. (New York Times article cited by Gawron & Kehler)

With expressions like *in that order*:
John, the kids, and Mary arrived to the summit in that order.
The persons who entered the room were John, Bill, Mary, and John, in that order.
The children arrived to the summit in the order they had reached the lake.
The children arrived to the summit in alphabetical order.
John sang, danced, and sang, in that order.

Sentences where *respectively* or *in that order* can be added with the same intended meaning:
The first three Fibonacci numbers are zero, one, and one (respectively / in that order). (Hewitt)
The top three teams in the Scottish premiership were Celtic, Rangers, and Hearts (respectively / in that order). (Hewitt)
The digits on my credit card are 3, 1, 4, 1, 5, 7, 1, 3, 9 (in that order).
The last Roland Garros winners are David, Øystein, Øystein, David, and Peter (in that order).

With the adjective *consecutive*:
a, b, c, and a are consecutive letters. (Florio)
The letters in the word "bad" are alphabetically consecutive.
The letters written by John and those written by Mary are alphabetically consecutive.

Cases where repetition matters, but not order
The mean of one, one, and two is one.
The sum of the numbers written by John is 17.

Cases where only "structuring" plays a role (potential superplural examples)
The square things, the blue things, and the wooden things overlap. (Linnebo & Nicolas)
In the context of a multi-participant video game with competing teams:
The boys, the girls, and the teachers played against each other. (Linnebo & Nicolas)

b) Lessons

What can be ordered?

Answer: not only single objects, but also "pluralities" (several objects at once):
John, the kids, and Mary arrived to the summit in that order.

Such pluralities are typically finite (e.g., the kids). But they can also be infinite:
The integers, the rationals, the reals, and the cardinals were invented by the Greeks, the Romans, the French, and the German, respectively.

How many different pluralities can be ordered?

In the usual cases, it is a finite number of pluralities that get ordered. e.g. above, four pluralities: the integers; the rationals; the reals; the cardinals.

What about sentences like these?
The natural numbers, in their usual order, are 0, 1, 2, ...
The cardinals, in their usual order, are 0, 1, 2, ..., $Aleph_0$, $Aleph_1$, $Aleph_2$, ...
Should they be treated in the same way as the other cases?

Are pluralities merely ordered? Or are they ranked?

In cases where there is no repetition, a mere order would be sufficient.
John, the kids, and Mary arrived to the summit in that order.
j < ks < m, where < is a partial plural relation

But as soon as repetition occurs, the pluralities must be ranked:
The last Roland Garros winners are David, Øystein, Øystein, David, and Peter, in that order.

So in general, sentences with *in that order* that involve several pluralities should invoke a ranking. The same is true with *respectively*:
The first three Fibonacci numbers are zero, one, and one, respectively.

What fixes or constrains the ordering/ranking?

When there is the conjunction *and*, the ranking is imposed by the order of words in the noun phrase or verb phrase:
John, the kids, and Mary arrived to the summit in that order.
Sue, Karen, and Bob jog, drive, and jog, respectively.
a, b, c, and a are consecutive letters.

With plural superlative noun phrases, their meaning fixes a ranking:
The three brightest students got the three highest grades, respectively. (Kay)

With definites, a comparison of temporal rankings can happen:
The children arrived to the summit in the order they had reached the lake.
Or general knowledge can tell us that a ranking exists:
The digits on my credit card [the successive digits of a number] *are 3, 1, 4, 1, 5, 7, 1, 3, 9, in that order.*
Or context and inference can supply a ranking:
The letters written by John and those written by Mary [as they appear on the board] *are alphabetically consecutive.*

So in many cases, the ordering cannot be retrieved from the surface order of constituents; instead meaning, general knowledge, context and inference are used to identify the relevant ranking.

II) Logico-philosophical accounts

a) Alternatives

Reference to some things in an order is logically primitive

Hewitt (2012) proposes that reference to some things in an order should be taken to be logically primitive. Not only can we refer to several things at once. We can also refer to several things in an order. The main predicate of the sentence then applies to these things in that order.

Multigrade predicates

Florio (manuscript) suggests that such sentences contain multigrade predicates, which can take varying numbers of arguments. Thus, in *Zero, one, and one are consecutive Fibonacci numbers*, the multigrade predicate *consecutive Fibonacci number* takes three arguments, while it takes only two in *Zero and one are consecutive Fibonacci numbers*:

consecutive-Fibonacci-numbers(0,1,1) vs consecutive-Fibonacci-numbers(0,1)

Similarly in other cases:

John, the kids, and Mary arrived to the summit in that order.	arrived-in-order(j,ks,m)
The mean of 1, 7, and 1 is 4.	mean(1,7,1) = 4
The boys, the girls, and the teachers played against each other.	played-against(bs,gs,ts)

Contextual orderings defined by partial plural predicates

Here, the semantic value of the plural expression remains its ordinary plural value: it refers to several things at once. But special expressions like *consecutive, respectively* and *in that order* operate on certain orderings or rankings identified in context, and defined by partial plural predicates.

Ex: *John, the kids, and Mary arrived to the summit in that order.*

Two orderings must be identified. The ordering associated with the subject, defined by the partial predicate F(i,xs), is identified given the ordering of words forming the subject. So we have: F(1,j) & F(2,ks) & F(3,m). The ordering associated with the verbal phrase, defined by the partial predicate G(i, xs), is identified as a temporal ordering. It specifies who arrived first to the summit, who arrived second, etc.

The sentence is true iff for all i and xs: F(i,xs) $\leftrightarrow$ G(i,xs).

Articulation of reference?

Ben-Yami (submitted) proposes that reference is articulated by syntax, and predicates are in general sensitive to this articulation of reference.

Ex: in the context of a multi-participant video game with competing teams:

The boys, the girls, and the teachers played against each other.

For Ben-Yami, the subject just refers to several things at once. But because the subject is syntactically complex, its referents are articulated into three "sub-pluralities". And the verbal predicate is sensitive to this articulation of reference.

According to Ben-Yami, every noun phrase articulates its referents in a certain way. Thus, only one "plurality" is articulated in *The candidates played against each other*.

Articulation of reference is always present and predicates are sensitive to it by default: every predicate comes with an extra-slot for articulation of reference. But in many cases, the articulation is actually not used when evaluating the sentence.

Conceivably, the articulation of reference could include the order of words, and so apply to cases like *John, the kids, and Mary arrived to the summit in that order.*

b) Pros and cons

Concerning primitive reference to things in an order

Should we take reference to things in an order to be logically primitive?
Hewitt argues that there are no good arguments for showing the contrary. But in my view, he hasn't established with positive arguments that it is logically primitive.

So can we manage without postulating that reference to things in an order is primitive?

Concerning multigrade predicates

In usual plural logic, *John and Paul lifted a piano together*: lifted(bs)

By contrast, under the multigrade view, 'lifted' is a predicate that takes a varying number of arguments: lifted(j,p)

Then what about: *The boys lifted a piano together.*
One might try this. *The boys* refers to several things at once. But it isn't the direct argument of the predicate. Instead, the plural term feeds distinct arguments to the multigrade predicate.

This seems odd, and moreover, this isn't applicable to all cases, since we can talk about innumerably many things: *The real numbers are positive or negative.* The multigrade approach seems to require finitely many arguments (or at least denumerably many).

A unified semantics of sentences with plural definites is preferable. So we should take them to always involve genuine plural predication:
The boys lifted a piano together: lifted(bs)

But then, if we want to maintain multigrade predication for conjunction, we need meaning postulates guaranteeing equivalences like these:
'bs' refers to John and Paul, 'j' refers to John, 'p' refers to Paul, so lifted(bs) iff lifted(j,p)

Once we have plural predication for plural definites, why should we keep multigrade predication? Possible answer: this provides an analysis of examples involving order, repetition, and "structuring" (cf. Florio):
John, the kids, and Mary arrived to the summit in that order. arrived-in-that-order(j,ks,m)

However, this works only if the arguments are fed to the multigrade predicate in a certain order and the predicate is sensitive to this. Then it seems that this must be a general feature of the multigrade approach. And in cases where order actually plays no role, special meaning postulates must tell us so. These complications are better avoided if possible.

Concerning contextual orderings

* *Motivation*

Usually, sentences with plurals are insensitive to order and repetition:
John, the kids, and Mary lifted the piano together.
Order and repetition play a role only in rare cases, when expressions like *in that order*, *respectively*, and *consecutive* are present.

So the following view is tempting. Plural expressions always retain their ordinary semantic value. But special expressions like *in that order* require the identification of certain orderings, which they compare. (Formally, these orderings could be seen as arguments of these expressions, arguments whose value is fixed in context.) When there is a conjunction, the ordering is specified by the surface order of constituents. But in other cases, meaning, general knowledge, context and inference must be used to identify the relevant orderings or rankings. A ranking is defined by a partial plural predicate of the form F(i,xs).

** Treatment of some other cases*

John, the kids, and Mary got gold, silver, and bronze, respectively.
One needs to identify an ordering for the subject, and one for the object of the verb. Since the subject is a conjunction of noun phrases, its associated ordering is specified by the order of the noun phrases. It's defined by a partial predicate F such that F(1,j) & F(2,ks) & F(3,m). Similarly, the ordering associated with the object is defined by a partial predicate such that G(1,g) & G(2,s) & G(3,b).
The sentence is true iff for all i, xs and ys, F(i,xs) & G(i,ys) → got(xs,ys).

The letters written by John [as they appear on the board] *are alphabetically consecutive.*
One needs to identify an ordering for the subject and compare it to the ordering expressed by the verbal predicate *are alphabetically consecutive*. Here, the ordering associated with the subject is contextually specified and characterized by a partial predicate F.
The sentence is true iff for all i, x and y, F(i,x) ∧ F(i+1,y) → consecutive(x,y).

What happens when order seems to play a role even without the presence of a special expression?
John, the kids, and Mary got gold, silver, and bronze (respectively).
Answer: plural sentences are typically ambiguous. This one is too, even if one interpretation is more salient. When *respectively* is added, this eliminates the ambiguity.
The digits on my credit card are 3, 1, 4, 1, 5, 7, 1, 3, 9 (in that order).
In that case, there is repetition and so something else seems required than regular plural ambiguity. Possible view: the expression *in that order* is silently present.

** Differences between contextual orderings* (C0) *and articulation of reference* (AR)
In common: plural expressions have their ordinary values, but something extra comes into play when evaluating the sentence.
Differences:
#AR: Articulation of reference is always present, but typically discarded.
CO: An ordering or ranking is present only when asked for by a special expression.

#AR: Articulation is postulated, but no truth-conditions have been given yet.
CO: Truth-conditions are given, and one can verify whether a given sentence is true or false.

#AR: Articulation of reference applies to cases involving repetition (*the mean of*) and "structuring" (superplurals); conceivably, it could also apply to cases involving order.
CO: Special expressions like *respectively*, *in that order*, and *consecutive* require the identification of orderings. To verify whether the sentence is true or false, one compares two orderings.
This isn't what happens in potential cases of superplural:
(In the context of a multi-participant video game with competing teams:)
The boys, the girls, and the teachers played against each other.
This doesn't involve order. If this is a genuine case of superplural, the predicate applies to a "superplurality", and no checking about the sub-pluralities is sufficient to establish whether the sentence is true or false. So we don't have to identify superplural cases with cases involving order. We can pursue a divide and conquer strategy, where each case is treated differently, according to its own needs.

III. The case of *respectively*

a) More data

Which expressions can be conjoined?

Kay (1989): *respectively* works with several kinds of conjoined expressions: noun phrases, verbs, verb phrases, adjectives, adverbs:

John and Mary talked to Bill and Sue, respectively.
John and Mary sang and danced, respectively.
The three top prize winners sang, danced and played the kazoo, respectively.
John's and Mary's manuscripts were competent and brilliant, respectively.
John and Mary were furious, volubly and icily, respectively.

Remark: it also seems to work with optional prepositional phrases:

John and Mary worked hard in London and in Paris, respectively.

Can any noun phrases be conjoined?

John and Mary talked to Bill and a student, respectively.
John and Mary talked to every boy and every girl, respectively.
John and Mary talked to each boy and more than three girls, respectively.

Conjunction is not required

Clarence, Florence, and Terrence got the three highest grades, respectively.
The three brightest students got the three highest grades, respectively. (Kay)

Respectively establishes a 1-1 mapping that must be based on independent rankings

Kay (1989):

Mr Smith and Mr Jones love Mrs Jones and Mrs Smith, respectively.

Respectively establishes a 1-1 mapping between the sets {Mr Smith, Mr Jones} and {Mrs Jones, Mrs Smith} and distributes the predicate *love* over the members of that mapping.

The mapping must be based on independent rankings of the members of the sets:

#*The boys love the girls, respectively.*
The three brightest students got the three highest grades, respectively.
These boys love these girls, respectively. (seems possible if the speaker has in mind some boys and some girls and who loves who)

The mapping need not be among atoms

The three women are, respectively, the mother and sisters of Randall Dale Adams, who has been [...] (New York Times, cited by Kay 1989)
John and Bill talked to the boys and the girls, respectively.
John and Bill talked to Mary, and Sue and Jenny, respectively.

What is the nature of the rankings?

With conjunction, it is order of mention:

John and Bill sang and danced.

With superlatives, their meaning fixes a ranking:

The three brightest students got the three highest grades, respectively.

The ranking can also be just contextual:

These boys love these girls, respectively.

This seems possible if the speaker has in mind some boys and girls and who loves who.

Similarly:
My students scored 95, 99 and 96, respectively.
This seems ok if the speaker has in mind some students and who scored what.

But out of the blue, this is out (Burnett p.c.):
#The three bright students scored 95, 99 and 96, respectively.

The ranking can be based on specialized background knowledge:
The horses finishing in the money were Augustus, Brutus and Cassius, respectively.
#The horses finishing out of the money were Augustus, Brutus and Cassius, respectively.

b) What are the interpretations of plural sentences without *respectively*?

Are they ambiguous or indeterminate?

This question should be raised in particular for these kinds of sentences:

i) Nominal conjunction [V nominal conjunction]

John and Bill talked to Mary and Sue.

And more generally: Plural NP [V plural NP]

The students hit the targets.

Gillon (1992) and Schwarzschild (1996) take them to be ambiguous. Verkuyl and van der Does (1995) take them to be indeterminate.

ii) Plural NP [predicate conjunction]

John and Bill are tall and short.
The students sang and danced.

"Distributive" and "cumulative" interpretations of predicate conjunction: ambiguity or indeterminacy

At the party, John and Bill sang and danced.
At least two kinds of verifying situations:
i) John sang and danced, and Bill sang and danced. ("distributive")
ii) One of John and Bill sang, the other danced. ("cumulative")
(Likewise for: *The students sang and danced*.)

Is there ambiguity or indeterminacy? In favor of ambiguity, this dialogue seems to make sense:
Sue: *At the party, John and Bill sang and danced.*
Helen: *That's not true: John didn't dance, he only sang.*
However, this dialogue seems strange:
Sue: *At the party, John and Bill sang and danced.*
Helen: *That's not true: they both sang and danced.*

What about positing instead the distributive interpretation *plus* an indeterminate interpretation, e.g. for *The students sang and danced*: the students participated in an event of singing and dancing, with nothing more specified concerning how each student participates? But then, wouldn't one prefer to have just the indeterminate interpretation?

When is the "cumulative interpretation" of predicate conjunction possible?

For Winter (2001), the predicates must be incompatible (*swim* and *fly*):
The ducks are swimming and flying. (some are swimming, some are flying)
The ducks are swimming and quacking. (all are swimming, all are quacking)
When the predicates are not incompatible (*swim* and *quack*), one would only get the "distributive interpretation": all the ducks are swimming and all the ducks are quacking.

However, this claim is too strong, for a "cumulative interpretation" does seem possible with compatible predicates:
At the party, John and Bill sang and danced, but I don't know who did what.
This means that the "distributive interpretation" is just strongly preferred when the predicates are compatible.

Similarly, with context (e.g. looks) and prosody, a "cumulative interpretation" of this sentence seems possible:
Mary and Sue are tall and good looking.

Order influences interpretation but does not fix it
The front and the back of the ship are called the bow and the stern, but which is which? (Chaves 2012)

c) The semantics of *respectively*: Chaves (2012)

Chaves focuses on two kinds of sentences:
i) Nominal conjunction [V nominal conjunction], *respectively.*
John and Bill talked to Mary and Sue, respectively.
ii) Nominal conjunction [verbal conjunction], *respectively.*
John and Bill sang and danced, respectively.

His basic position is this. The sentences without the adverb are ambiguous. When the adverb is present, this imposes a semantic constraint that selects one interpretation.

The first kind of sentence, without *respectively*, is more generally of the form:
Plural NP [V plural NP]
The students hit the targets.
Like Scha (1981), Gillon (1992) and many others, Chaves takes sentences of this kind to be ambiguous between many interpretations. Adopt a suitable semantics predicting this ambiguity. When the adverb *respectively* is present, this adds a constraint (the 1-1 mapping) relating the orderings given by order of mention, context or background knowledge, and this selects one interpretation.

The second kind of sentence, without *respectively*, is more generally of the form:
Plural NP [predicate conjunction]
The students are tall and short.
Chaves takes them to have two kinds of readings, which were mentioned earlier:
John and Bill sang and danced.
- John sang and danced, and Bill sang and danced. ("distributive")
- One of John and Bill sang, the other danced. ("cumulative")

He assumes that verbs come with neo-Davidsonian eventuality arguments. I find his complete formal account problematic. But here is, in essence, how it renders the cumulative reading:
$\exists e_1 \exists e_2 \exists x_1 \exists x_2 (x_1 \neq x_2 \wedge j+b = x_1+x_2 \wedge \text{sing}(e_1, x_1) \wedge \text{dance}(e_2, x_2))$
When *respectively* is present, order of mention orders the semantic values of the nominal conjuncts ($j < b$) and the semantic values of the predicate conjuncts ($e_1 < e_2$). The adverb requires that the values given to x_1 and x_2 respect these orders, and so $x_1 = j$ and $x_2 = b$.

Remark 1: Chaves just orders the semantic values of nominals and predicates. This isn't enough in case of repetition of nominals:
The four best poems were written, respectively, by John, Bill, John, and Mary.
The first three Fibonacci numbers are zero, one, and one, respectively.
The general solution is to use rankings, where each semantic value is given a rank.

Remark 2: I find Chaves' detailed formal account problematic. But his basic position makes a lot of sense. He advocates, for *respectively*, the same kind of position as I do more generally for various order-sensitive expressions. For Chaves, sentences without the adverb are ambiguous. When *respectively* is present, this selects one interpretation, in which there is a 1-1 mapping between the rankings supplied by order of mention, context or background knowledge.

Remark 3: If one thinks the sentences are not ambiguous but indeterminate, then the adverb doesn't select one interpretation, but simply narrows the truth-conditions to those fitting the independently supplied rankings.

References

Ben-Yami, H. Submitted. Superplurals versus articulated reference.

Chaves, R. 2012. Conjunction, cumulation and respectively readings. Journal of Linguistics 48(2), 297-344.

Florio, S. Manuscript. Superplurals and lists.

Gawron, J.M. & Kehler, A. 2004. The semantics of respective readings, conjunction and filler-gap dependencies. Linguistics and Philosophy 27, 169-207.

Gillon, B. S. 1992. Towards a common semantics for English count and mass nouns. Linguistics and Philosophy 15: 597-639

Hewitt, S. 2012. The logic of finite order. Notre Dame Journal of Formal Logic. 53(3), 297-318.

Kay, P. 1989. Contextual operators: respective, respectively, and vice versa. In K.Hall, M.Meacham & R.Shapiro (eds.), Proceedings of the 15th Annual Meeting of the Berkeley Linguistics Society, 181-192. BLS.

Linnebo, Ø. & Nicolas, D. 2008. Superplurals in English. Analysis 68(3), 186-197.

Schwarzschild, R. 1996. Pluralities. Dordrecht: Kluwer Academic Publishers

Verkuyl, H. and J. van der Does. 1995. The semantics of plural noun phrases. In J. van der Does & J. van Eijk (eds), Quantifiers, logic, and language. CSLI.

Winter, Y. 2001. Flexibility principles in Boolean semantics. MIT Press.