The Logic of Mass Expressions

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Mass expressions: *water, gold, furniture, silverware...*
Count expressions: *cat, table, army, committee...*

1. What are mass expressions?
1.1 Syntactic criteria
Traditional, dominant view (Weinreich 1966, Krifka 1991, Gillon 1992):
Common nouns are divided in two morphosyntactic subclasses, mass and count nouns.
Mass nouns:
- are invariable in grammatical number
- can be used with determiners like *much* and *a lot of*

Count nouns:
- can be used in the singular and in the plural
- can be employed with numerals like *one* and determiners like *many*

1.2 Semantic criteria
Cumulative reference (Quine 1960):
Let *M* be a mass noun. Suppose we can truly say of something x that *This is M,* and of something distinct, y, that *This is M* (with *this* now referring to y). Then in the same circumstance, we can also refer to x and y together, and say of x and y that *This is M*.
(Plural count nouns also have the same property.)

Distributive reference (Cheng 1973, Lønning 1987, Higginbotham 1994):
Suppose that we can truly say of something x that *This is M*. Then in the same circumstance, for anything y that is part of x, we can also truly say of y that *This is M* (with *this* now referring to y).
Problem: many mass nouns don't seem to refer distributively (Gillon 1992, Nicolas 2002a):
- Microscopic parts: water is made of oxygen and hydrogen, but oxygen isn't water.
- Macroscopic parts: a table is furniture, a leg of the table is part of the table, but the leg isn't furniture.

We now consider several approaches to the semantics of mass nouns, seeking to identify what is good (and bad) in each, in order to get closer to a satisfying overall treatment.

2. The purely mereological approach (Moravcsik 1973)
- Uses mereological sums as the denotata of mass nouns.
- Interprets mass predication (e.g. *This is water*) in terms of parthood.

Suppose there is some water in a bottle, a, and some water in a cup, b. Then *the water in the bottle and the cup* denotes the sum of a and b, noted $a \vee b$. More generally, we can sum all the portions of water together. This sum is what *water* denotes.

This is M is true iff [*this*] ≤ [*M*]
where […] is the denotation function, [*this*] is the sum of what is demonstrated, and [*M*] is the sum of all M

Ex: *This is water* is true iff [*this*] $\leq$ [*water*], the sum of all water.
Problem with parts that are too small to count as M (oxygen for *water*, the leg of a table for *furniture*).

Moravcsik (1973) proposes two solutions. The first immediately fails (cf. Bunt 1985).
Second proposal: put restrictions on the part-whole relation. Let *M* be a mass noun, [*M*] the sum it denotes, and $\leq_M$ the associated part-whole relation.
This is M is true iff [*this*] $\leq_M$ [*M*]
But this doesn't account for the validity of certain syllogisms (Burge 1972):
This is gold. Gold is metal. Therefore, this is metal.
[*this*] $\leq_{GOLD}$ [*gold*] & [*gold*] $\leq_{METAL}$ [*metal*] $\rightarrow$ [*this*] $\leq_{METAL}$ [*metal*]
This is invalid, since only a uniform part-whole relation could guarantee transitivity.

The 'WOOD = FURNITURE' problem for the mereological approach (Parson 1970):
Suppose all wood is used to make up furniture, and all furniture is made of wood.
Then, the sum of the wood seems to be identical with the sum of the furniture.
Therefore, *The wood P* and *The furniture P* must have the same truth-value, for any predicate *P*. Yet, *The furniture is heterogeneous* may be true, while *The wood is heterogeneous* is false.

Remark 1: Instead of classical extensional mereology, one could use a less constrained join semi-lattice, with parthood defined in terms of the join $\vee$: $x \leq y =_{def} x \vee y = y$

Remark 2: Thus, one may want to deny that the wood is identical to the furniture. Indeed, if the furniture is broken, it ceases to exist, while the wood does not. (Cf. also a ship and the wood it is made of, a man and its molecules...) Parsons' argument is based on a controversial metaphysical assumption, which the semantics need not make.

Remark 3: The purely mereological approach faces yet another, very general problem. One still needs a uniform framework for doing semantics: for proper names, singular count nouns, plurals, mass nouns, adjectives, verbs, etc. It seems that this has to be set theory, or something as powerful like "non-singular" or "plural" logic.

3. The purely set-theoretic approach (Burge 1972, Grandy 1973, Montague 1973)
- Mass nouns are ordinary predicates denoting sets.
- Mass predication is interpreted as set membership.

This is M is true iff [*this*] $\subseteq$ [*M*]
Some M P is true iff [*M*] $\cap$ [*P*] $\neq \varnothing$
where [*this*] is the set whose elements are what is demonstrated, [*M*] is the set having for elements everything that is M, [*P*] is the set having for elements everything that P

Problem with definite descriptions, especially in statements of identity over time:
The clay that was on the desk on July 1st is identical with the clay that was on the table on July 2nd.
(Context of utterance: three solid bits of clay were on the desk on July 1st, and two solid bits of clay were on the table on July 2nd. Example inspired by Cartwright 1965.)
Which set could make [*the clay that was on the desk on July 1st*] = [*the clay that was on the table on July 2nd*] true?
What about the set of all minimal parts of clay? However, what a given mass noun applies to may be indefinitely divisible. Semantics should not force mass nouns to have minimal parts (cf. Bunt 1985, Gillon 1992).

4. The mixed set-theoretic and mereological approach
- Mass predication (as in *This is water*) should not be understood in terms of parthood, but in terms of set membership.
- The denotation of a mass noun *M* (the set whose elements are everything that is M) should be the join semi-lattice generated by the sum or join operation on portions of M.

This is M is true iff [*this*] ⊆ [*M*]
Some M P is true iff [*M*] ∩ [*P*] ≠ ∅
The M (that Q) P is true iff [*the M (that Q)*] ⊆ [*P*]
where [*this*] is the set having for sole member the sum of what is demonstrated, [*M*] is the set of everything that is M, [*the M (that Q)*] is the set having for sole member the sum of everything that is some M (that Q), [*P*] is the set having for members everything that P

This mixed approach keeps what is good in each approach.
Decisive advantage over the purely mereological approach: the overall framework for doing semantics remains the usual one, set theory. (Gillon (1992) and Nicolas (2010) propose such a mixed view, with an additional component, "coverings".)

5. Negation
A difficulty appears with negation (Roeper 1983, Lønning 1987, Higginbotham 1994).

Consider *The M P* and its negation *The M not P*.
For instance: *The gold is in the safe* and *The gold is not in the safe.*
Suppose that: [*gold*] = {a,b,a∨b}, [*the gold*] = {a∨b}, and [*in the safe*] = {a}
According to the mixed view:
The gold is in the safe is true iff [*the gold*] ⊆ [*in the safe*]
So the sentence is false.
Moreover, this equivalence seems plausible: *The M not P* is true iff *The M P* is false.
So *The gold is not in the safe* is predicted to be true. This is a problem for the mixed approach developed so far, since one would want to ascribe the same status to the positive sentence and its negation. Either because both sentences are taken to be false. Or because both are judged inapplicable in the cricumstance, being as it were partly true and partly false.

Consider also *the gold that is in the safe* and *the gold that is not in the safe*.
Intuitively, the first noun phrase designates the solid bit of gold a, while the second designates b.
However, under the mixed approach, *The gold is not in the safe* is true. So it would seem that a+b is in [*is not in the safe*], in [*gold that is not in the safe*], and in [*the gold that is not in the safe*].

Solution proposed by Roeper (1983), Lønning (1987) and Higginbotham (1994):
Mass nouns and their predicates denote elements of a Boolean algebra: (B,≤,∨,∧,0,1,-)
Predication is understood in terms of Boolean intersection:
This is M is true iff [*this*] ∧ [*M*] = [*this*] iff [*this*] ≤ [*M*]
The M P is true iff [*M*] ∧ [*P*] = [*M*] iff [*M*] ≤ [*P*]
Some M P is true iff [*M*] ∧ [*P*] ≠ 0
where [*this*] is the join of what is demonstrated, [*M*] is join of everything that is M, and [*P*] is the join of everything that P

Negation is defined in terms of Boolean complement: [*not P*] = -[*P*]
The M not P is true iff [*M*] ≤ [*not P*] = -[*P*]
Example: [*gold*] = a∨b = 1, [*is in the safe*] = a, [*is not in the safe*] = -a = b
So *The gold is in the safe* and *The gold is not in the safe* are predicted to be false.

Moreover: [*M that P*] = [*M*] ∧ [*P*]
So [*gold that is in the safe*] = [*gold*] ∧ [*is in the safe*] = (a∨b)∧a = a
And [*gold that is not in the safe*] = (a∨b)∧b = b

Remark 1: The whole universe of discourse (for mass nouns and their predicates) is specified by a single Boolean algebra. Predication is defined in terms of Boolean intersection. This works with mass nouns and predicates that refer distributively.

But mass nouns like *furniture* clearly don't. Nor does a predicate like *made by John*. So the Boolean approach may ascribe incorrect truth-conditions to *This is furniture*, *Some furniture is made by John*, and *The furniture is made by John*. For instance, [*this*] ≤ [*furniture*] does not guarantee that what is demonstrated is furniture

The Boolean approach doesn't seem to work in these cases, even though the same difficulties with negation appear. So an appropriate solution had better not be tied to the assumption of distributive reference.

Remark 2: Indeed, the treatment of negation can be adapted within the mixed approach, without requiring distributive reference.

Basic idea: if something x P and something y not P, then x and y do not overlap.
So let's define [*not P*] as the set comprising anything that does not overlap ∨[*P*], the sum of everything that P.

Remark 3: However, there are many predicates whose negation cannot be defined in terms of Boolean complement or non-overlap.

Consider the vague predicate *cheap*. This could be the case:
[*furniture*] = {a,b,a∨b}, [*cheap*] = {a,b}, [*not cheap*] = {a∨b}
So [*cheap*] and [*not cheap*] overlap.

And there can also be overlap with an exact predicate like *costs fifty euros.*

So non-overlap should not be required for negation. In general, [*not P*] cannot be defined in terms of [*P*]. Instead, [*P*] and [*not P*] must be separately specified.

Remark 4: The same difficulties also appear with plurals: just replace *gold* by *pieces of furniture* and *in the safe* by *made by John* in the examples above.

There is no agreement on what is the proper treatment of negated plural sentences.
Still, this is a popular view (Krifka 1996, Löbner 2000; see Breheny 2005 a contrario):
Sentences like *The pieces of furniture are in the safe* and its negation make a presupposition of "indivisibility": they can be used felicitously only if all the pieces of furniture are in safe or if none is.

6. Quantifiers

What is the semantics of quantifiers combining with mass nouns: *some, all, no, only, little, much, most, two liters of*...?

Higginbotham and May (1981) use generalized quantification for quantifiers combining with count nouns (*some, all, no, only, few, many, most, two*…).

Inspired by Roeper (1983) and Lønning (1987), Higginbotham (1994) applies similar ideas to mass nouns. His proposals are made in the Boolean approach just criticized. So I transpose them directly into the mixed set-theoretic and mereological framework.

We focus on sentences *Q M P*, where *Q* is a quantifier, *M* a mass noun, *P* a predicate. [*M*] is the denotation of the mass noun, i.e. the set that has for members everything that is M (a join semi-lattice). [*P*] is the set that has for members everything that P.

Some M P is true iff $[M] \cap [P] \neq \emptyset$
All M P is true iff $[M] \cap [P] = [M]$
No M P is true iff $[M] \cap [P] = \emptyset$
Only M P is true iff $[M] \cap [P] = [P]$
where $\cap$ is set intersection
(Examples: *Some / All / No / Only gold was stolen*.)

With the other quantifiers (*little*, *much*, *most*, *two liters of*…), one seems to say something about the quantity of M (*little gold*). So let us suppose that a mass noun *M* has an associated function measuring quantity, μ, which is monotonic:
$x \leq y \rightarrow \mu(x) \leq \mu(y)$
and additive:
$\neg\exists z\,(z \leq x \;\&\; z \leq y) \rightarrow \mu(x \vee y) = \mu(x)+\mu(y)$

One can also define the measure of a set E:
$\mu(E) =_{def} \mu(\vee E)$, where $\vee E$ is the sum (or join) of the elements of E

Little$_1$ *M P* is true iff $\mu([M] \cap [P]) \leq p$
Little$_2$ *M P* is true iff $\mu([M] \cap [P]) \leq r*\mu([M])$
Much$_1$ *M P* is true iff $\mu([M] \cap [P]) \geq q$
Much$_2$ *M P* is true iff $\mu([M] \cap [P]) \geq s*\mu([M])$
Most M P is true iff $\mu([M] \cap [P]) \geq \mu([M]) / 2$
Two liters of M P is true iff $\mu([M] \cap [P]) = 2$, with a function μ measuring in liters
The numerical values *p*, *q*, *r* and *s* are specified contextually when a sentence is uttered.

Above, *little* and *much* are given an "absolute" meaning and a "proportional" one.
Thus, *Much gold was stolen* may mean that:
- The stolen gold was a large quantity of gold (absolute interpretation):
$\mu([M] \cap [P]) \geq q$, where *q* is specified contextually
- The stolen gold was a great proportion of the gold (proportional interpretation):
$\mu([M] \cap [P]) \geq s*\mu([M])$, where *r* is specified contextually

Remark 1: This general framework leaves room for improvement concerning the specific meaning(s) attributed to a given quantifier. For instance, Solt (2009) argues in favor of a different condition for the quantifier *most*.

Remark 2: In the case of the count quantifiers *few* and *many*, there is evidence that each quantifier is really ambiguous between two interpretations (Partee 1989). Is there is similar evidence in the case of *little* and *much*?

Remark 3: Adding negation to this picture creates the same difficulties as we saw above in the case of definites.

7. Collective and non-collective construals, coverings

According to Gillon (1992, 1996), a sentence containing a mass noun may receive so-called "collective", "distributive" and "intermediate" construals, modulo the meaning of the lexical items composing the sentence, context of speech and knowledge of the world.

This silverware costs a hundred euros.
- Collective construal: the silverware costs, altogether, a hundred euros.
- Distributive construal: each piece of silverware, by itself, costs a hundred euros.
- An intermediate construal: there are two sets of silverware, each costing a hundred euros.

This fruit was wrapped in that paper.
- An intermediate construal with respect to each argument of the verb: there are several pieces of paper, each enclosing several pieces of fruit.

These cases correspond to partitions of the denotation of the mass noun phrase(s).

However, some interpretations correspond to a more general notion of "covering":
A set X is a covering of a set Y just in case the (mereological) sum of the elements of X is identical with the sum of the elements of Y.

This livestock carried that furniture.
- An intermediate construal: some pieces of furniture were repeatedly part of the furniture carried by some of the livestock.
The relation of carrying applies between elements of a covering of [*this livestock*] and elements of a covering of [*that furniture*].

So it seems that the semantics of mass nouns should leave room, not only for partitions, but also for all kinds of coverings.

Gillon's account, slightly modified technically:
The denotation of a mass noun *M* is set [*M*] that has for elements everything that is M.

This is M is true iff [*this*] $\subseteq$ [*M*]
where [*this*] is the set having for sole element the sum of what is demonstrated

A set Y is an *M*-covering of a set Z just in case these two conditions are satisfied:
- Y is a subset of [*M*]: Y $\subseteq$ [*M*]
- The sum of the elements of Y is identical to the sum of the elements of Z

The interpretation of sentences like the following depends on the choice of an *M*-covering of the noun's denotation. Relative to this choice of covering C:
The M P is true iff $C \subseteq [P]$ (ex: *The silverware costs a hundred euros*)
Some M P is true iff $C \cap [P] \neq \varnothing$
All M P is true iff $C \cap [P] = C$

Gillon doesn't extend his account to other quantified statements. However, this is easily done, following section 6:
$\mu(E) =_{def} \mu(\vee E)$, where $\vee E$ is the sum of the elements of E
Most M P is true iff $\mu(C \cap [P]) \geq \mu(C) / 2$
And similarly for the other quantifiers whose interpretation involve a measure.

8. Non-singular logic

There are many similarities between the semantics of mass nouns and plurals, cf. sections 5, 6 and 7.

Also, at a very intuitive level, if there are eight pieces of silverware on the table, then the speaker seems to refer to eight things at once when he says:

The silverware that is on the table comes from Italy.

If this intuition is taken seriously, then a mass noun isn't a singular term. Rather, it is a non-singular term that may refer to one or several things at once.

Nicolas (2008) puts forwards a semantics of mass nouns based on this intuition. It is cashed out in "non-singular" or "plural" logic.

In usual logic frameworks, like predicate logic, constants and variables are singular in the following sense. Under any interpretation, a constant is interpreted as one individual, and under any assignment, a variable is interpreted as one individual.

By contrast, "non-singular" or "plural" logic possesses singular and non-singular constants and variables. Under any interpretation and variable assignment, a non-singular term (a constant or a variable) may be interpreted as one or more individuals in the domain of interpretation. In particular, a formula consisting of a predicate whose argument is a non-singular constant is true just in case the constant is interpreted as one or more individuals that jointly satisfy the predicate. (See Linnebo 2008 for an overview of "plural" logic.)

The resulting semantics has the following features:

- Unrestricted composition (the existence of mereological sums) is replaced by "non-singular" or "plural" reference. (Cf. also Nicolas 2009.)
- Combined with a generalized notion of covering, this allows a treatment of identity statements different from that offered by the mixed set-theoretic and mereological approach (where what remains identical over time is a certain mereological sum).

Non-singularist notion of covering for a mass noun M

The css are an M-covering of the as just in case the following three conditions are satisfied:

i) Any thing among° the css is M:

$y \angle^{\circ} css \rightarrow My$

ii) For any "plurality" of the css, no two members of the "plurality" overlap:

$xs \angle css \rightarrow \neg(\exists u\, \exists v\, (u \angle xs \wedge v \angle xs \wedge u \neq v \wedge Ouv))$

iii) For anything v, v overlaps some thing among° the css just in case v overlaps one of the as:

$\exists y\, (y \angle^{\circ} css \wedge Ovy) \leftrightarrow \exists w\, (w \angle as \wedge Ovw)$

The clay that was on the desk on July 1st is identical with the clay that was on the table on July 2nd.

Context of utterance: three solid bits of clay, the as, were on the desk on July 1st, and two solid bits of clay, the bs, were on the table on July 2nd.

In effect, the sentence will be true when a common covering can be chosen for the as and the bs. This will mean that there are some small bits of clay, each of which has retained its identity over time. On July 1st, these bits of clay were so arranged that they made up the as (i.e., they were a covering of the as). On July 2nd, they were arranged differently, in such a way that they made up the bs.

NB: This doesn't require the existence of minimal parts. Only the existence of a common division.

References

Breheny, R. (2005). Exhaustivity, homogeneity & definiteness. Amsterdam Colloquium Vol. 15, 59-65. Amsterdam: Universiteit van Amsterdam.

Bunt, H. C. (1985). Mass terms and model-theoretic semantics. Cambridge: Cambridge University Press.

Burge, T. (1972). Truth and mass terms. Journal of Philosophy 69, 263-282.

Carlson, G. (1977). Reference to kinds in English. PhD, MIT.

Cartwright, H. (1965). Heraclitus and the bath water. Philosophical Review 74, 466-485.

Cheng, C.-Y. (1973). Comments on Moravcsik's paper. In Hintikka et al. (Eds.), Approaches to natural language, 286-288. D. Reidel Publishing Company.

Chierchia, G. (2010). Mass nouns, vagueness and semantic variation. Synthese 174(1), 99-149.

Cocchiarella, N. (2009). Mass nouns in the logic of classes as many. Journal of Philosophical Logic 38, 343-361.

Damourette, J. & Pichon, E. (1927) Des mots à la pensée. Essai de grammaire de la langue française. Paris: d'Artey.

Gillon, B. S. (1992). Towards a common semantics for English count and mass nouns. Linguistics and Philosophy 15, 597-639.

Gillon, B. S. (1996). Collectivity and distributivity internal to English noun phrases. Language Sciences 18, 443-468.

Grandy, R. E. (1973). Comments on Moravscik's paper. In Hintikka et al. (eds.), Approaches to natural language, 295-300. D. Reidel Publishing Company.

Higginbotham, J. (1994). Mass and count quantifiers. Linguistics and Philosophy 17, 447-480.

Higginbotham, J. & May, R. (1981). Questions, quantifiers and crossing. Linguistic Review 1, 41-80.

Krifka, M. (1991). Massennomina. In A. von Stechow & D. Wunderlich (Eds.), Semantik, ein internationales Handbuch, 399-417. Berlin: de Gruyter.

Krifka, M. (1996). Pragmatic strengthening in plural predications and donkey sentences. In T. Galloway & J. Spence (eds.), SALT VI. Cornell University Press.

Landman, F. (2011). Count nouns, mass nouns, neat nouns, mess nouns. In B. H. Partee, M. Glanzberg & J. Skilters (eds.), Formal semantics and pragmatics. Discourse, context and models. The Baltic International Yearbook of Cognition, Logic and Communication, Vol. 6 (2010). Manhattan, KS: New Prairie Press.

Laycock, H. (2006). Words without objects. Oxford: Clarendon Press.

Linnebo, Ø. (2008). Plural quantification. In E. N. Zalta (Ed.), The Stanford Encyclopedia of Philosophy (Spring 2008 Edition).

Löbner, S. (2000). Polarity in natural language. Linguistics and Philosophy 23, 213-308.

Lønning, J.T. (1987). Mass terms and quantification. Linguistics and Philosophy 10, 1-52.

Moltmann, F. (2007). Event, tropes and truth making. Philosophical Studies 134(3), 363-403.

Montague, R. (1973). The proper treatment of mass terms in English. Reprinted in J. F. Pelletier (ed.) (1979), Mass terms, 137-166. D. Reidel Publishing Company.

Moravcsik, J. (1973). Mass terms in English. In Hintikka et al. (eds.), Approaches to natural language, 263-285. D. Reidel Publishing Company.

Nicolas, D. (2002a). La distinction entre noms massifs et noms comptables. Aspects linguistiques et conceptuels. Louvain: Editions Peeters.

Nicolas, D. (2002b, unpublished.) Conversions of count nouns into mass nouns in French: the roles of semantic and pragmatic factors in their interpretations. Retrieved from http://d.a.nicolas.free.fr/Nicolas-Conversions.pdf

Nicolas, D. (2004a). Is there anything characteristic about the meaning of a count noun? Revue de la Lexicologie 18-19.

Nicolas, D. (2004b). The semantics of nouns derived from gradable adjectives. In C. Meier & M. Weisgerber (eds.), Proceedings of Sinn und Bedeutung 8 (http://web.uni-frankfurt.de/fb10/SuB8).

Nicolas, D. (2008). Mass nouns and plural logic. Linguistics and philosophy 31(2), 211-244.

Nicolas, D. (2009). Mereological essentialism, composition, and stuff: a reply to Kristie Miller. Erkenntnis 71(3), 425-429.

Nicolas, D. (2010). Towards a semantics of mass expressions derived from gradable expressions. Recherches Linguistiques de Vincennes 39, 163-198.

Parsons, T. (1970). An analysis of mass and amount terms. Reprinted in J. F. Pelletier (ed.) (1979), Mass terms, 137-166. Dordrecht: D. Reidel Publishing Company.

Partee, B. H. (1989). Many quantifiers. In J. Powers & K. de Jong (eds), Proceedings of the 5th Eastern States Conference on Linguistics, 383-402. Columbus: Ohio State University.

Pelletier, J. F. (1975). Non-singular reference: some preliminaries. Reprinted in J. F. Pelletier (ed.) (1979), Mass terms, 1-14. Dordrecht: D. Reidel Publishing Company.

Pelletier, J. F. & L. Schubert. (2003). Mass expressions. In D. Gabbay & F. Guenthner (eds.), Handbook of philosophical logic, 2nd edition, Volume 10, 249-336. Dordrecht: Kluwer Academic Publishers.

Quine, W. V. O. (1960). Word and object. Cambridge: MIT Press.

Roeper, P. (1983). Semantics for mass terms with quantifiers. Noûs 17, 251-265.

Schwarzschild, R. (1996). Pluralities. Dordrecht: Kluwer Academic Publishers.

Solt, S. (2009). The semantics of adjectives of quantity. PhD, City University of New York.

ter Meulen, A. (1981). An intensional logic for mass terms. Philosophical studies 40, 105-125.

Ware, R. X. (1975). Some bits and pieces. Reprinted in J. F. Pelletier (ed.) (1979), Mass terms, 15-29. D. Reidel Publishing Company.

Weinreich, U. (1966). Explorations in semantic theory. In Current trends in linguistics Vol.3, 395-477. The Hague: Mouton de Gruyter.